D

Subject Code
વિષય કોડ :- 21023

Que. Paper Series
પ્રશ્નપુસ્તિકા કોડ

Mathematics
(Science)

Candidate's Roll No.

Supervisor's Signature

Time : 1 Hours
Total Ques. : 50
Total Marks : 50

1202298420

Do not open the Question Booklet until ask to do so.

Instructions to Candidates / ઉમેદવારોને સૂચના

(1) There are **50** questions in the Test Booklet. The answer of each question is any **one** out of **A, B, C** and **D**. Four alternatives are given with the question. You have to answer all the questions.

(2) Each correct answer carries **(1)one** mark. For each wrong multiple answer for each question will be treated as wrong. No negative marking for wrong answer.

(3) You have to answer on the OMR Sheet is given separately to you. **For example** : Which State of India has the longest Sea Coast ?

(A) Maharashtra (B) Tamil Nadu
(C) Gujarat (D) Andra Pradesh

In OMR Answer Sheet :

Ⓐ Ⓑ ● Ⓓ

The true answer is **"C"**. Hence circle of **"C"** is blackened (encode).

(4) Don't write anything on the Test Booklet.

(5) Use **blue/black inked ballpoint pen** for filling responses in the OMR Answer Sheet. Any other ink, pen or Pencil is strictly prohibited.

(6) During exam from candidate, if any false matter, guide, cheats, slips, any handwritten material, any printed material, mobile phone, pager, calculator or any electronic equipments are found he/she will be disqualified.

(7) Gossips, making noise or disobeying instructions given by Invigilator will be considered disobedience.

(8) Do Rough Work on last page.

(1) આ પ્રશ્નપુસ્તિકામાં કુલ **50** પ્રશ્નો છે. પ્રત્યેક પ્રશ્નનો સાચો ઉત્તર **A, B, C** અને **D** પૈકી કોઈ **એક** છે. પ્રશ્નની સાથે જ ચારેય વિકલ્પો આપવામાં આવેલા છે. તમારે બધા જ પ્રશ્નોના ઉત્તર આપવાના છે.

(2) પ્રત્યેક પ્રશ્નના સાચા ઉત્તર માટે **1(એક)** ગુણ છે. એક કરતા વધુ વિકલ્પ ડાર્ક કરનાર જવાબને ખોટો ગણવામાં આવશે. ખોટા જવાબ માટે કોઈ ગુણ કપાત થશે નહીં.

(3) ઉત્તર આપવા માટે **અલગ ઉત્તરવહી (OMR SHEET) આપી છે.**
ઉત્તર નીચેના **ઉદાહરણ** પ્રમાણે આપવાના છે.
ભારતનું કયું રાજ્ય સૌથી લાંબો દરિયા-કિનારો ધરાવે છે ?

(A) મહારાષ્ટ્ર (B) તામિલનાડું
(C) ગુજરાત (D) આંધ્રપ્રદેશ

ઉત્તરવહી (OMR SHEET)માં

Ⓐ Ⓑ ● Ⓓ

ઉપરોક્ત પ્રશ્નનો સાચો ઉત્તર **"C"** છે. આથી **"C"**નું વર્તુળ કાળું (encode) કરેલ છે.

(4) આ પ્રશ્નપુસ્તિકામાં કશું જ લખવાનું નથી.

(5) ઉત્તરવહીમાં ઉત્તરો **વાદળી / કાળી શાહીની બોલપોઈન્ટ** પેનથી આપવાનાં છે. અન્ય શાહી, પેન કે પેન્સિલનો ઉપયોગ કરી શકાશે નહીં.

(6) પરીક્ષા દરમ્યાન ઉમેદવાર પાસેથી કોઈ પણ સાહિત્ય, ગાઈડ, માર્ગદર્શિકા, કાપલી, સ્લીપો, અન્ય હસ્તલિખિત કે પ્રિન્ટેડ સાહિત્ય, મોબાઈલ ફોન, પેજર, કેલ્ક્યુલેટર કે અન્ય વીજાણુ ઉપકરણો હોવાનું જણાશે તો ઉમેદવારને ગેરલાયક ગણવામાં આવશે.

(7) ચાલુ પરીક્ષા દરમ્યાન અંદરોઅંદર ગુસપુસ કરવી, અવાજ કરવો કે નિરીક્ષકની સૂચનાઓનું ઉલ્લંઘન કરવું તે ગેરશિસ્ત ગણાશે.

(8) રફકામ છેલ્લાં પેજ પર કરવું.

ઉત્તરવહી (OMR Answer sheet) બે પ્રતમાં છે. પરીક્ષા પૂરી થયા બાદ ઉત્તરવહી (OMR Answer Sheet)ની પ્રથમ સ્કેનીંગ પ્રત (Scaning Copy) વર્ગ નિરીક્ષકને પરત કર્યા બાદ જ વર્ગખંડ છોડવાનો રહેશે. તેમ કરવામાં કસૂર થયેથી શિસ્તભંગનાં પગલાં ગણી પરીક્ષા માટે જે તે ઉમેદવારને ગેરલાયક ઠેરવવામાં આવશે. બીજી ઉમેદવાર પ્રત (Candidate Copy) ઉમેદવાર સાથે લઈ જઈ શકશે. પ્રશ્નપુસ્તિકા અને બીજી પ્રત ઉમેદવાર પોતાની જોડે લઈ જઈ શકશે.

1 Consider the function $f(x, y) = |x| + |y|$

(1) f is continuous at 0
(2) f is differentiable at 0
(A) The statements (1) and (2) are true
(B) The statements (1) and (2) are false
(C) Only the statement (1) is true
(D) Only the statement (2) is true

2 If $f_n(x) = nxe^{-nx^2}$ for n = 1, 2, 3,... and real x, then

(A) $\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 \lim_{n\to\infty} f_n(x)dx$

(B) $\lim_{n\to\infty} \int_0^1 f_n(x)dx \neq \int_0^1 \lim_{n\to\infty} f_n(x)dx$

(C) $\lim_{n\to\infty} f_n(x)$ does not exist

(D) None of these

3 Let $X = \mathbb{R}^2$ with usual metric space. $A = \left\{\left(x, \frac{1}{x}\right); x \in \mathbb{R}\right\}$ and $B = \{(x, 0) : x \geq 0\}$.

(A) A and B are not compact
(B) A and B are compact
(C) A is compact and B is not compact
(D) A is not compact and B is compact

4 Which of the following are homoeomorphic spaces with usual metric on $\mathbb{R}^2$? Consider $X = \mathbb{R}^2$,

$D_2 = \{(x, y) : x^2 + y^2 < 1\}$, $sq^2 = \{(x, y) : \max(|x|, |y|) < 1\}$

(A) X, D_2 and sq^2 are homoeomorphic
(B) X, D_2 and sq^2 are not homoeomorphic
(C) D_2 and sq^2 are not homoeomorphic
(D) X and sq^2 are homoeomorphic

5 What is the largest possible value of $f(0)$, where $f(x)$ is continuous and differentiable on the interval [–5, 0], such that $f(-5) = 8$ and $f'(c) \leq 2$?

(A) 2
(B) –2
(C) 18
(D) –18

6 If $f(x)$ is continuous and differentiable over [–2; 5] and $-4 \le f'(x) \le 3$ for all x in (–2; 5), then the greatest possible value of $f(5)-f(-2)$ is

(A) 7 (B) 21

(C) 15 (D) 9

7 If $f(z)$ and $\overline{f(z)}$ both analytic function in a domain D then

(1) $f(z)$ is constant in D.

(2) $\overline{f(z)}$ is constant in D.

(A) The statements (1) and (2) are true

(B) Only the statement (1) is true

(C) Only the statement (2) is true

(D) Both statements (1) and (2) are not true

8 The equation $y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$ is

(A) Hyperbolic Type

(B) Parabolic Type

(C) Elliptic Type

(D) None of these

9 The unit digit of the number 2^{2022} is

(A) 2 (B) 4

(C) 8 (D) 6

10 How many ways can 3 Mathematics students, 3 Statistics students, 2 Physics students and 2 Chemistry students sit around a circular table such that students from the same subject sit together ?

(A) 864 (B) 684

(C) 846 (D) 876

11 The number of non-zero solutions of $z^2 + 2\bar{z} = 0$ is

(A) 2

(B) 3

(C) 4

(D) 5

12 The number of values of a for which the equation $x^3 - x + a = 0$ has double root is

(A) 0

(B) 1

(C) 2

(D) Infinite

13 Let A be matrix of order 2023 × 2023. If the rank of A is 2021, then rank of adjoint of A is

(A) 2021

(B) 2

(C) 1

(D) 0

14 The number of roots of $g(x) = 5x^4 - 4x + 1$ in [0, 1]

(A) 0

(B) 1

(C) 2

(D) 3

15 For the equation $|x|^2 + |x| - 6 = 0$

(A) There is only one root

(B) The sum of the roots is –1

(C) The sum of the roots is 0

(D) The product of the roots is –6

16 The value of $\lim_{n\to\infty} \frac{1\cdot 1!+2\cdot 2!+3\cdot 3!+....+n\cdot n!}{(n+1)!}$

(A) 1 (B) 2

(C) $\frac{1}{2}$ (D) Does not exist

17 The solution of $\frac{dy}{dx}=3^{x+y}$ is

(A) $3^x-3^{-y}=c$ (B) $3^{-x}+3^{-y}=c$

(C) $3^{-x}-3^{y}=c$ (D) $3^x+3^{-y}=c$

18 Suppose $A=\begin{pmatrix}2 & 1 & 0\\ 1 & b & d\\ 1 & b & d+1\end{pmatrix}$, $X=\begin{pmatrix}x\\ y\\ z\end{pmatrix}$, $B=\begin{pmatrix}f\\ g\\ h\end{pmatrix}$. Which of the following is condition on A and U such that the system $AX = U$ has no solution ?

(A) $g-\frac{1}{2}f-d(h-g)\neq 0$

(B) $g-\frac{1}{2}f-d(h-g)=0$

(C) $g+\frac{1}{2}f+d(h-g)\neq 0$

(D) $g-\frac{1}{2}f+d(h+g)\neq 0$

19 The number of common solutions of $x^{36}-1=0$ and $x^{24}-1=0$ in the set of complex number is

(A) 1 (B) 2

(C) 6 (D) 12

20 Let $f:\mathbb{R}\to\mathbb{R}$ be a function such that $f(x-y)=f(x)f(y)$ and $f(x)\neq 0$ for all x. What is the value $f(3)$?

(A) 1 (B) 2

(C) 3 (D) 4

21 If $I = \int_0^1 \frac{1}{1+x^8}\,dx$, then

(A) $I < \frac{1}{2}$

(B) $I < \frac{\pi}{4}$

(C) $I > \frac{\pi}{4}$

(D) $I = \frac{\pi}{4}$

22 How many five-digit positive integers that are divisible by 3 can be formed using the digits 0, 1, 2, 3, 4 and 5 without any of the digits getting repeated ?

(A) 216

(B) 96

(C) 120

(D) 625

23 The differential equation of the family of parabolas having their vertices at the origin and their foci on the X-axis is

(A) $2xdy - ydx = 0$

(B) $xdy + ydx = 0$

(C) $2ydx - xdy = 0$

(D) $dy - xdx = 0$

24 The number of primes p such that p, $p + 10$, $p + 14$ are all prime numbers is

(A) 0

(B) 1

(C) 3

(D) Infinitely many

25 The number of solutions of the equation $\sqrt{1-\sin x} = \cos x$ in $[0, 5\pi]$ is equal to

(A) 3

(B) 6

(C) 8

(D) 11

26 What is the residue of $\frac{\cos z}{z^2}$ at 0 ?

(A) $2\pi i$ (B) 1

(C) $\frac{1}{2}$ (D) 0

27 If $y_1(x) = x$ is a solution of $(1-x)y'' + xy' - y = 0,\ x > 1$, then what is its general solution ?

(A) $y(x) = c_1 x + c_2 e^x$

(B) $y(x) = c_1 x + c_2 x^{-1} e^x$

(C) $y(x) = c_1 x + c_2 x^2$

(D) $y(x) = c_1 x + c_2 (x-1)$

28 Which of the following is the general solution of $y'' + y = x^4 + x^2$?

(A) $y(x) = c_1 \cos x + c_2 \sin x + x^4 - 11x^2 + 22$

(B) $y(x) = c_1 \cos x + c_2 \sin x + x^4 - 11x^2 + 24$

(C) $y(x) = c_1 e^x + c_2 e^{-x} - x^4 - 11x^2 - 24$

(D) $y(x) = c_1 e^x + c_2 e^{-x} + x^4 - 11x^2 - 22$

29 Let M be set of all 3×3 matrices A with real entries. For any $A = (a_{ij}),\ B = (b_{ij})$ belonging to M. Which of the following are metric on M ?

(A) $d_1(A, B) = \max\left\{\left|a_{ij} - b_{ij}\right| : i, j = 1, 2, 3\right\}$

(B) $d_2(A, B) = \max\left\{\sum_{j=1}^{3}\left|a_{ij} - b_{ij}\right| : i = 1, 2, 3\right\}$

(C) $d_3(A, B) = \begin{cases} 0, & A = B \\ 1, & A \neq B \end{cases}$

(D) $d_3(A, B) = \min\left\{\sum_{j=1}^{3}\left|a_{ij} - b_{ij}\right| : i = 1, 2, 3\right\}$

30 The number of subgroups of $\mathbb{Z}_{12}$ are _____.

(A) 5 (B) 4

(C) 12 (D) 11

31 Which of the following is not true ?

(A) There exists a field of order 9.

(B) There exists a field of order 11.

(C) There exists a field of order 13.

(D) There exists a field of order 15.

32 Let R be the ring of all 2×2 matrices with integer entries. Which of the following subsets of R is an integral domain ?

(A) $\left\{\begin{pmatrix} 0 & x \\ y & 0 \end{pmatrix} : x, y \in \mathbb{Z}\right\}$ (B) $\left\{\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} : x, y \in \mathbb{Z}\right\}$

(C) $\left\{\begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} : x \in \mathbb{Z}\right\}$ (D) $\left\{\begin{pmatrix} x & y \\ y & z \end{pmatrix} : x, y, z \in \mathbb{Z}\right\}$

33 If f_1 and f_2 are real valued convex functions define on [0, 1], which of the following is/are convex ?

(A) $f_1 + f_2$ (B) $f_1 - f_2$

(C) $f_1 \times f_2$ (D) $\frac{f_1}{f_2}$ if $f_2 \neq 0$

34 Consider the ordinary differential equation $y'' + P(x)y' + Q(x) = 0$, where P and Q are smooth function. Let y_1 and y_2 be any two solutions of the ODE. Let $W(x)$ be the corresponding Wronskian. Then which of the following is always true ?

(A) If y_1 and y_2 are linearly dependent then there exists x_1 and x_2 such that $W(x_1) = 0$ and $W(x_2) \neq 0$.

(B) If y_1 and y_2 are linearly independent then $W(x) = 0$ for all x.

(C) If y_1 and y_2 are linearly dependent then $W(x) \neq 0$ for all x.

(D) If y_1 and y_2 are linearly independent then $W(x) \neq 0$ for all x.

35 Let J denote a 101×101 matrix with all the entries equal to 1 and I denoted the identity matrix of order 101. Then the determinant of $I - J$ is

(A) 101 (B) 100

(C) 0 (D) –100

36 Let A, B be $n \times n$ matrices such that $BA + B^2 = I - BA^2$, where I is the $n \times n$ identity matrix. Which of the following is always true ?

(A) A is nonsingular
(B) B is nonsingular
(C) $A + B$ is nonsingular
(D) AB is nonsingular

37 Which of the following statements is FALSE ?

(A) The set of rational numbers is an abelian group under addition.
(B) The set of integers is an abelian group under addition.
(C) The set of rational numbers form an abelian group under multiplication.
(D) The set of real numbers excluding zero is an abelian group under multiplication.

38 The function $f(z) = \dfrac{1 - \cot z}{z}$ has

(1) The zero of order 1 at $2\pi + \dfrac{\pi}{4}$

(2) The pole of order 1 at 0

(A) The statements (1) and (2) are true.
(B) The statements (1) and (2) are false.
(C) Only the statement (1) is true.
(D) Only the statement (2) is true.

39 Given an $n \times n$ matrix B defined by $e^B = \sum_{n=0}^{\infty} \dfrac{B^j}{j!}$. Let p be the characteristic polynomial of B then the matrix $e^{p(B)}$ is

(A) $I_{n \times n}$
(B) $0_{n \times n}$
(C) $e \times I_{n \times n}$
(D) $\pi \times I_{n \times n}$

40 Consider the following statements

(1) Any two groups of order 4 are isomorphic.
(2) Every abelian group is cyclic.

(A) The statements (1) and (2) are true.
(B) The statements (1) and (2) are false.
(C) Only the statement (1) is true.
(D) Only the statement (2) is true.

41 The series $\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^{20}}, x \in \mathbb{R}$ is converges

(A) only for $x = 0$

(B) uniformly only for $x \in [-\pi, \pi]$

(C) uniformly only for $x \in \mathbb{R} \setminus \{n\pi : n \in \mathbb{Z}\}$

(D) uniformly for all $x \in \mathbb{R}$

42 Given that $(a_n)_{n\geq 1}$ a sequence of real numbers, which of the following statements is true ?

(A) $\sum a_n$ converges, then $\sum a_n^4$ converges

(B) $\sum |a_n|$ converges, then $\sum a_n^2$ converges

(C) $\sum |a_n|$ converges, then $\sum a_n^3$ converges

(D) $\sum |a_n|$ diverges, then $\sum a_n^2$ converges

43 Let A be an 2023 × 2023 matrix such that the set of all its nonzero eigen values has exactly 2020 elements. Which of the following statements is true?

(A) Rank (A) $\leq$ 2020

(B) Rank (A) $\leq$ 2022

(C) Rank (A) $\geq$ 2020

(D) A^2 has 2020 distinct nonzero elements

44 Let λ, μ be distinct eigen values of matrix A order 2. Then which of the following statements must be true ?

(A) A^2 has distinct eigen value

(B) A^n is not a scalar multiple of identity for any integer n.

(C) $\lambda + \mu$ is eigen value of A.

(D) A has two linearly independent eigen vectors

45 Which of the following subspace W of $\mathbb{R}^2$ is invariant under $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(x; y) = (x + y; x + y)$?

(A) $W = \{(x, y) : x + 3y = 0\}$ (B) $W = \{(x, y) : x + y = 0\}$

(C) $W = \{(x, y) : x - y = 0\}$ (D) $W = \{(x, y) : x - y = 1\}$

46 Let $A, B \in M_n(\mathbb{R})$. If A and B are nilpotent, which of the following is TRUE ?

(A) $A + B$ is invertible
(B) $A + B$ is nilpotent
(C) $A + B$ is nilpotent if $AB = BA$
(D) $A + B$ is invertible if $AB = BA$

47 Consider the matrix $A = \begin{bmatrix} 1^2 & 2^2 & \dots & n^2 \\ 2^2 & 3^2 & \dots & (n+1)^2 \\ \vdots & \vdots & \vdots & \vdots \\ n^2 & (n+1)^2 & \dots & (2n-1)^2 \end{bmatrix}$ for $n \geq 4$. Then the determinate of A is

(A) $n + 1$
(B) n
(C) n^2
(D) 0

48 If $T \in M_4(\mathbb{R})$ and T is similar to the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 10 \end{bmatrix}$.

Let $B = T^2 + T$ and $p(\lambda) = \lambda^2 + \lambda$, then trace of B is

(A) $1^2 + 5^2 + 8^2 + 10^2$
(B) $1 + 5 + 8 + 10$
(C) $\sum_{i=1}^{4} p(i)$
(D) $p(1) + p(5) + p(8) + p(10)$

49 The interval of convergence of the series $\sum_{n=0}^{\infty} n!(5x+1)^n$

(A) $x = -\frac{1}{5}$
(B) Empty set
(C) $\left(-\frac{1}{5}, \frac{1}{5}\right)$
(D) $\left(-\frac{1}{5}, \frac{1}{5}\right]$

50 Let $f(x, y) = (x^2 + y^2)\sin\left(\frac{1}{x^2 + y^2}\right)$, $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$.

Then

(1) f is not differentiable.

(2) $f_x(0; 0)$ and $f_y(0; 0)$ do not exist.

(A) The statements (1) and (2) are true
(B) The statements (1) and (2) are false
(C) Only the statement (1) is true
(D) Only the statement (2) is true

SPACE FOR ROUGH WORK